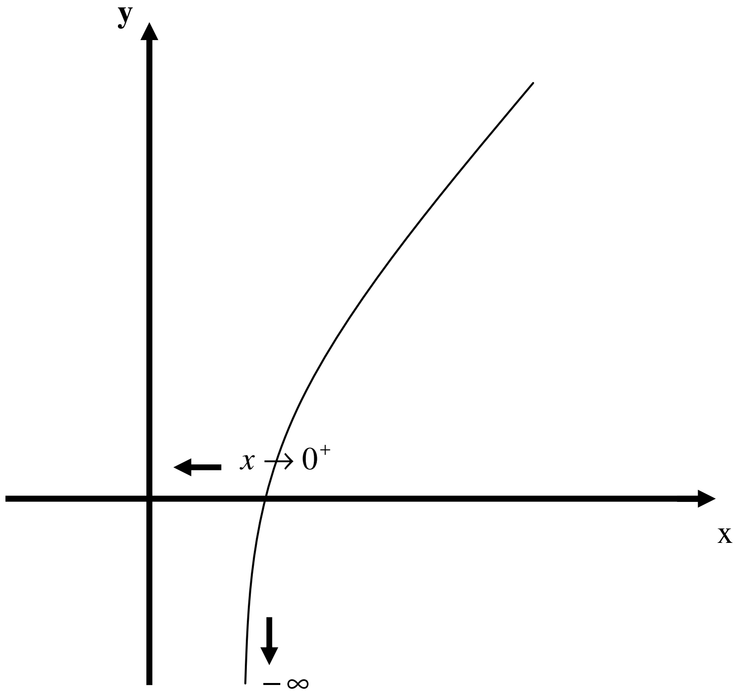
Cambiamento di variabile

Esempio: $\lim_{x\to 0^+} \frac{\log^2 x+1}{\log x+2\log^2 x} = \frac{(-\infty)^2+1}{-\infty+2(-\infty)^2} = \frac{\infty}{-\infty+\infty} = IND.$



Il limite è per $x \to 0^+$ perché la funzione non esiste per $x \to 0^-$

$\log x = z$ se $x \to 0^+$
$z \to -\infty$

$$\lim_{z\to -\infty} \frac{z^2+1}{z+2z^2}$$

Osservazione:
Dopo il cambiamento di variabile non è necessario sostituire z con $-\infty$ perché l'indeterminazione resta,quindi si procede con la *messa in evidenza.*

tende a 0

$$\frac{z^2\left(1+\frac{1}{z^2}\right)}{z^2\left(\frac{1}{z}+2\right)} = \frac{1}{2}$$

tende a 0

Esercizi

1. $$\lim_{x\to\infty}\frac{3x^2-x+2}{2x^2+1}=\frac{3(\infty)^2-\infty+2}{2(\infty)^2+1}=\frac{\infty-\infty}{\infty}=IND$$

$$\lim_{x\to\infty}\frac{x^4\left(3-\frac{1}{x^3}+\frac{2}{x^4}\right)}{x^2\left(2+\frac{1}{x^2}\right)}=\lim_{x\to\infty}\frac{x^2\left(3-\frac{1}{x^3}+\frac{2}{x^4}\right)}{\left(2+\frac{1}{x^2}\right)}=\frac{\infty^2\left(3-\frac{1}{\infty^3}+\frac{2}{\infty^4}\right)}{\left(2+\frac{1}{\infty^2}\right)}=\frac{\infty}{2}=\infty$$

2. $$\lim_{x\to\frac{\pi}{3}}\frac{sen2x}{(1-\cos x)^2}=\lim_{x\to\frac{\pi}{3}}\frac{sen2\frac{\pi}{3}}{\left(1-\cos\frac{\pi}{3}\right)^2}=\frac{sen120^\circ}{(1-\cos 60^\circ)^2}=\frac{\frac{\sqrt{3}}{2}}{\left(1-\frac{1}{2}\right)^2}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{4}}=\frac{\sqrt{3}}{2}\bullet 4=2\sqrt{3}$$

3. $$\lim_{x\to 0^+}\frac{1+\log x}{2-\log x}=\frac{1-\infty}{2+\infty}=\frac{\infty}{\infty}=IND$$

$z=\log x$ $\qquad x\to o$ $\qquad z\to\infty$

$$\lim_{x\to 0^+}\frac{1+z}{2-z}=\frac{z\left(\frac{1}{z}+1\right)}{z\left(\frac{2}{z}-1\right)}=\frac{0+1}{0-1}=\frac{1}{-1}=-1$$

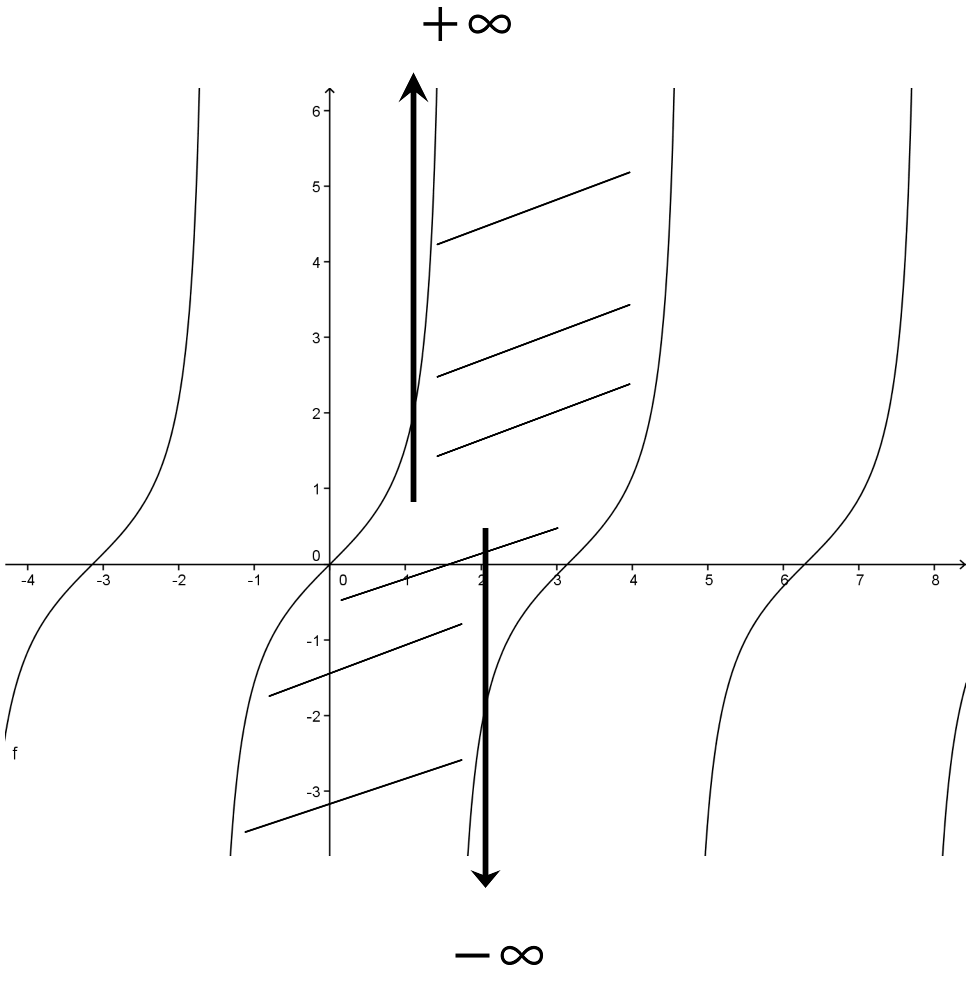
4. $$\lim_{x\to\frac{\pi}{2}}\log\frac{3+tg^2x}{2+tg^2x}=\log\frac{3+tg^2 90}{2+tg^2 90}=\log\frac{3+\infty}{2+\infty}=\log\frac{\infty}{\infty}=IND$$

$tgx=0$ $\qquad x\to\frac{\pi}{2}$ $\qquad z\to\infty$

$$\lim_{x\to\infty}\log\frac{3+z}{2+z}=\lim_{x\to\infty}\log z\frac{z\left(\frac{3}{z}+1\right)}{z\left(\frac{2}{z}+1\right)}=\log 1=0$$

5. $\lim_{x\to+\infty} tg\left(\frac{\pi}{2}+e^{-x}\right)=tg\left(\frac{\pi}{2}+e^{-\infty}\right)=tg\left(\frac{\pi}{2}+\frac{1}{\infty}\right)=tg(90+0)=tg(90^{+})=-\infty$

$e=2,7172...$



6. $\lim_{x\to-\infty} arctg\,\frac{e^{x}+1}{e^{x}+\sqrt{3}}=\lim_{x\to-\infty} arctg\,\frac{e^{-\infty}+1}{e^{-\infty}+\sqrt{3}}=arctg\,\frac{1}{\sqrt{3}}=30^{\circ}$

$e^{-\infty}=\frac{1}{e^{+\infty}}=\frac{1}{\infty}=0$